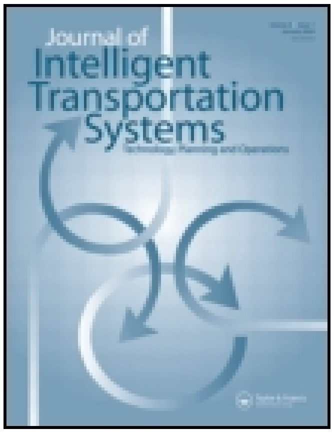

Journal of Intelligent Transportation Systems: Technology, Planning, and Operations

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gits20

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Accepted author version posted online: 28 Oct 2013.Published online: 24 Nov 2014.

Journal of Intelligent Transportation Systems, 19(1):63–77, 2014

ISSN: 1547-2450 print / 1547-2442 online
DOI: 10.1080/15472450.2013.856721

Micro-Simulation Model for Assessing the Risk of Vehicle–Pedestrian Road Accidents

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Data on traffic accidents clearly point to road black spots, where the accident rate is always high. However, road safety research is still far from understanding why these particular places on a road are risky. The reason is the lack of sufficient knowledge on how pedestrians and drivers interact when facing a potentially dangerous traffic situation, and the lack of an integrated framework that relates the data on human behavior to real-world traffic situations. We attempt to tackle this problem by developing SAFEPED, a multi-agent microscopic three-dimensional (3D) simulation of vehicle and pedestrian dynamics at a black spot. SAFEPED is a test platform for evaluating experimentally estimated drivers' and pedestrians' behavioral rules, and estimating accident risks in different traffic situations. It aims to analyze the design of existing and future black spots and to assess alternative architectural and environmental solutions in order to identify maximally efficient safety countermeasures.

Keywords Agent-Based Modeling; Black Spot; Spatially Explicit Modeling; Traffic Accidents

INTRODUCTION

Micro-Simulation of Road Accidents: From a Static to a Dynamic View

Accident statistics reveal factors of risk and establish the dependencies of accident rates on the characteristics and parameters of roads, vehicles, pedestrians, traffic, and the environment at an accident location (Campbell, 2003; Chang, 2008; Vlahogianni et al., 2012; Zegeer et al., 2005). However, statistical models are inherently static, and are unable to reflect the sequence of events that cause an accident (Gettman & Head, 2003). Their power for assessing and comparing the effects of possible changes in the infrastructure or traffic conditions at the dangerous location is thus inherently limited. Studies of black spots—seemingly regular road locations with unexpectedly high and stable accident rates—confirm that identifying and resolving safety threats requires an individual solution for each spot (Gitelman et al., 2010).

Traffic engineers' intuition is often sufficient for treatment of problems related to common problems of black spots of a certain type. Successful examples include the installation of the several hundred countdown signals at pedestrian crossings in San Francisco that reduced the number of pedestrian injuries caused by crashes with vehicles by 52% (Markowitz et al., 2006), and the system for detecting pedestrians approaching a crosswalk zone that warns drivers of pedestrian presence (Hakkert et al., 2002). In both examples, the effectiveness of the safety measures was evaluated by comparing the accident rates before and after the implementation of safety measures.

However, failures in reducing accident rates at sites that have been modified are often not reported. Black spots do not cease to be black spots and traffic engineers still lack the tools for assessing proposed safety measure changes to traffic regulations and infrastructure of a specific spot (Gitelman et al., 2010). Safety measures are costly to implement and may be successful for one spot and unsuccessful for the other.

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Color versions of one or more of the figures in the article can be found online at www.tandfonline.com/gits.

Dynamic simulation modeling of traffic accidents at a black spot can provide a solution to the problem of safety measure assessment. Using the model, the chain of events that caused an accident can be investigated, and the reasons for and consequences of risky behavior of drivers and pedestrians can be understood. This article presents a pilot version of such a model.

Field Studies of the Accident Microdynamics

During the last decade, a series of large-scale studies aimed at developing reliable indicators of vehicle precrash conditions was performed within the framework of the Intelligent Transportation Systems (ITS) program of the U.S. Department of Transportation. The research focused on "last second" urgent maneuvering, and resulted in significant amounts of data, collected during real-time observations of driver behavior and car movement (Kiefer et al., 2003; Klauer et al., 2006a, 2006b), as well as during simulator-based driving (Najm & Smith, 2004; Smith et al., 2003). The on-road data include kinematic characteristics of vehicles, real-time measurements of the distance to other objects, and video of drivers' behavior. In parallel, laboratory experiments provided meaningful data on drivers' behavior in potential accident scenarios, such as a lane-change maneuver (Malta et al., 2009) or unexpected behavior of leading vehicles (Smith et al., 2003).

Recent computer-based analysis of videos of road locations provided essential knowledge on driver and pedestrian behavior on the road. Video data provide the basis for modeling driver and pedestrian decision making, for example, for estimating the probability that a pedestrian would decide to cross the road depending on the distance to an approaching car, its velocity, and the road geometry (Sun et al., 2003; Papadimitriou et al., 2009; Wang et al., 2010; Prato et al., 2012; Robin et al., 2010). These models serve as building blocks for the dynamic model of driver–pedestrian interactions on the road in preaccident and accident situations.

Micromodeling of Vehicle–Pedestrian Conflict

The majority of micromodels of vehicle–pedestrian conflict describe pedestrian decision making in respect to the potential danger of a vehicle-related accident. Typically, logit models of a pedestrian's choice between two alternatives—to cross the road or to wait until the car passes—are constructed, in which the probability of crossing is related to the approaching vehicle time gap, age of the pedestrian, the number of pedestrians crossing in a group, and so on. Models of this kind provide very likely predictions of pedestrians' decisions (Schroeder & Rouphail, 2011). Extensions of this approach include motorists' yielding behavior (Papadimitriou et al., 2009; Sun et al., 2003) and pedestrians' jaywalking outside of crossing facilities (Wang et al., 2010).

Detailed representation of several patterns of pedestrian motion is studied in Robin et al. (2009). The cross-nested logit model includes submodels for five patterns of pedestrian behavior—constrained (collision avoidance, leader following), and unconstrained (maintaining direction, turning toward destination, free flow acceleration). The model represents the change in these patterns with respect to changes of motion speed and direction.

Discreet choice models, however, do not address evolution of conflict after the pedestrian, driver, or both have made their decisions. This can be done with the dynamic modeling of vehicle–pedestrian interactions, which is still underdeveloped but has proved its usefulness in wide range of transportation applications, such as the simulation of large-scale transportation systems (Balmer et al., 2004), of traffic signal control (El-Tantawy et al., 2014), and of pedestrian–vehicle dynamics in emergency situations (Zhang & Chang, 2013). Typically, the dynamic models focus on either vehicle or pedestrian traffic and avoid combining these two flows within the same model. The major reasons are inherent behavioral differences between pedestrians and drivers concerning route choice and compliance with traffic regulations. Advanced models of vehicle traffic, such as ArchiSim (Doniec et al., 2008; Ksontini at al., 2012), or VISSIM, PARAMICS, SUMO, and Aimsun (Ishaque & Noland, 2008) focus on the vehicle flow while utilizing an intentionally simplified view of pedestrians. Models of pedestrian crowd dynamics specify pedestrian interactions but ignore details of vehicle traffic (Papadimitriou et al., 2009).

The model of pedestrians' disobedience to traffic laws at crosswalks (Zhang & Duan, 2007) is a rare example of a dynamic model of vehicle–pedestrian interactions. It combines the Nagel–Schreckenberg Cellular Automata (CA) model of vehicle flow (Nagel & Schreckenberg, 1992) and the pedestrian submodel. The model demonstrates that while the total pedestrian volume is the most important factor affecting the capacity of mixed traffic at crosswalks, occasional lawbreakers additionally decrease traffic capacity. However, to represent vehicle flow, the model partitions space into square cells of the size of several meters. These cells are too large for adequate microscopic representation of pedestrian motion in accidents.

In this article, we present SAFEPED—a high-resolution, spatially explicit, dynamic simulation system for assessing the safety of traffic environments at pedestrian crossings. SAFEPED combines a continuous representation of space with the simulated motion of drivers and pedestrians that utilizes comprehensive robotic techniques of obstacle avoidance. We apply this innovative approach for a universal description of mixed vehicle/pedestrian motion in a continuous space for the study of road safety at black spots. SAFEPED is a spatially explicit agent-based model that represents traffic spot infrastructure and moving objects in fine three-dimensional (3D) details, and operates at a high time resolution of 1/20th of a second. Behavioral rules of SAFEPED agents—vehicles and pedestrians—are based, when possible, on the experimental data.

SAFEPED, AN AGENT-BASED MODEL OF VEHICLE-PEDESTRIAN INTERACTIONS

Agent-based (AB) techniques are especially convenient for modeling vehicular–pedestrian conflict (Benenson & Torrens, 2004; Benenson et al., 2005). Direct simulation of the behavior of agents (vehicles and pedestrians) acting within a precise 3D environment enables identifying risk factors and investigating the effectiveness of proposed safety measures. Scenarios with different numbers of vehicles and pedestrians of various kinds and behaviors can be investigated. Agents' actions and their outcome, for example, accidents, can be recorded and analyzed. Results of experiments on the behavior of vehicles and pedestrians can be directly interpreted in terms of agents' behavioral rules. SAFEPED motion behavior rules exploit robotic approaches to real-time motion planning and maneuvering in a dynamic environment (Fiorini & Shiller, 1998). The rules account for basic imperfections of human visual perception, limitations in pedestrian locomotion, and car mobility.

The 3D Representation of Black Spots

SAFEPED is built on a precise 3D representation of a traffic spot's surface and infrastructure, including road borders, parked vehicles, pedestrian crossings, buildings, trees, traffic lights and signs. Figure 1 shows the trajectories of agents, with different types of lines indicating the type of agent[1] and agents' priorities in terms of the right of way.

SAFEPED Agents and Their Behavior

SAFEPED drivers and pedestrians behave autonomously, according to a set of probabilistic behavioral rules. Each agent—driver or pedestrian—is assigned with an agent's profile that includes height, width, velocity, steering, and acceleration/deceleration capabilities.

Agents' Motion at a Macro Level

Each SAFEPED agent tries to maintain its desired predefined velocity, and aims to follow its predefined trajectory, as shown in Figure 1. That is, we do not consider the entire sequence of the behavioral decisions of pedestrians and drivers, but start simulations from the moment when all agents participating in the scene have already decided to move or to wait and have chosen the trajectory of movement. However, in the vast majority of situations, it is impossible to follow a predefined trajectory because of other moving and/or stationary objects. In this case, driver and pedestrian agents react, not necessarily adequately, to the behavior of the other autonomous agents that they are able to identify. The agents then decide whether to deviate left or right from their predefined trajectory, to accelerate, to decelerate, or even to stop. The agents can then decide to return to their predefined trajectory (if road conditions permit), or to continue along a new trajectory.

SAFEPED makes it possible to set decision-making priorities that reflect traffic rules and agreements at every intersection of agents' trajectories. SAFEPED assigns a priority level to each agent, and when two agents approach the point of intersection of their trajectories and notice each other, they both know that the agent with the higher priority will react before the other one. For example, in Figure 1, pedestrians have higher priorities when crossing the road than vehicles, and vehicles driving along the main street have priority over vehicles merging from the right (highlighted parts of trajectories). Note that the priority system includes the case when the higher priority agent decides that the other agent is moving too fast, and decides to slow down or stop and give way to the other agent. If the trajectories of two agents intersect and priorities are not assigned, both agents know there are no priorities, and the agent reacting first is selected randomly.

Agents' Microbehavior in Conflict Situations

Road safety in dynamic environments requires motion planning where vehicles and pedestrians should avoid dynamic and static obstacles. Motion planning and obstacle avoidance in robotics use velocity space (VS), also known as the velocity obstacle (VO), instead of the standard 3D "configuration space." The problem of avoiding one or many mobile or immobile obstacles is treated in the velocity space. In our model, vehicle and pedestrian agents follow a robotic motion-planning algorithm for dynamic environments as originally proposed in (Fiorini & Shiller, 1998). Further improvements algorithms include the Reciprocal Velocity Obstacle (Snape et al., 2011) and the Hybrid Reciprocal Velocity Obstacle (Van den Berg et al., 2008). The Reciprocal Velocity Obstacle algorithms incorporate the reactive nature of all agents by allowing each agent to assume that the other agents involved in the scene reciprocate by taking measures to avoid collision, instead of one agent taking all the responsibility for avoiding a collision (as is the case in the traditional velocity obstacle). Drivers and pedestrians aim to follow their predefined trajectories (different for each agent) with minimal deviation. A critical advantage of these algorithms is their applicability to a set of objects that essentially vary in their inherent velocities, in our case vehicles and pedestrians. Another critical advantage of all velocity obstacle algorithms is the time horizon. This parameter defines the possible future locations of the agents based on their current location and velocity. Time horizon has a major effect on the possible safe trajectories that avoid a crash. Various ranges of time horizon can be easily applied and tested in SAFEPED.

The basic algorithm considers the velocity obstacle (VO)—the set of all velocities of a moving object that will

[1]The figures in this article are shown in black and white (B/W), while the actual model view of a spot is in realistic colors.

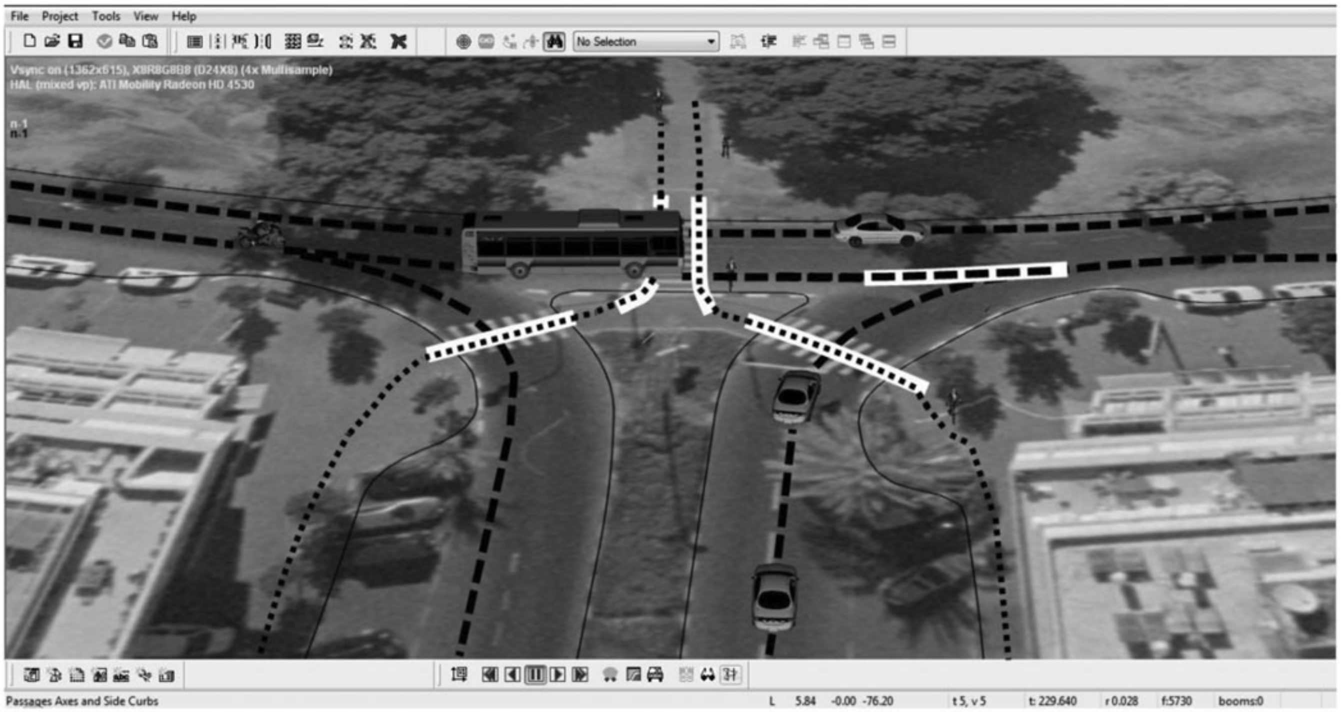


Figure 1 SAFEPED model scene showing agents' trajectories. Dashed and dotted lines represent vehicle and pedestrian trajectories, respectively. Highlighted intervals represent the areas of possible interactions between vehicles and pedestrians and the priorities.

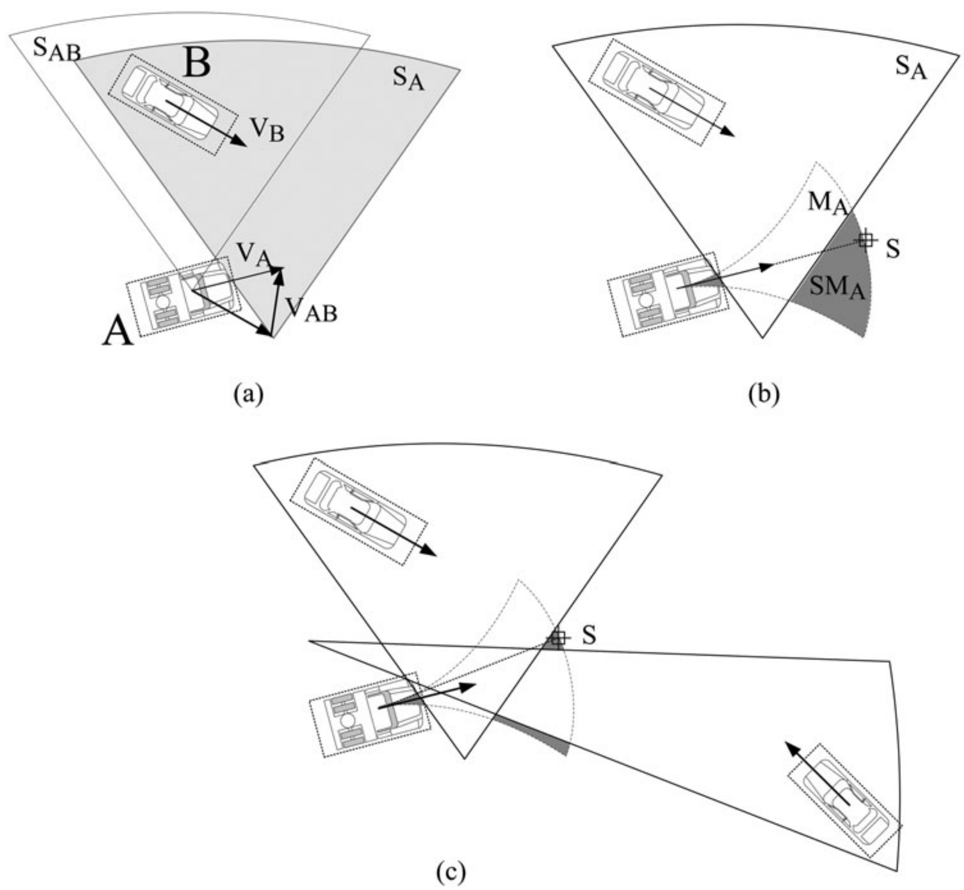


Figure 2 An example of the avoidance maneuver algorithm as implemented in the SAFEPED according to Fiorini and Shiller (1998), where more details can be found.

result in a collision with another moving object at some moment in time, assuming that the other objects maintain their current velocity. In our model, the concept of VO is applied for computation of avoidance maneuvers, including steering, accelerating, and decelerating (Figure 2).

In Figure 2a, car A is moving at a velocity V_A, and car B at V_B. Car A is trying to avoid collision with car B by changing its velocity (speed and/or direction), while car B maintains its original velocity. The white sector (S_{AB}) in Figure 2a, defined by Fiorini and Shiller (1998) as the *collision cone*, denotes the set of relative velocities V_{AB} of car A relative to car B that would result in a collision within a time horizon Δt. In other words, any relative velocity vector of car A relative to car B within S_{AB} would result in a collision if car B maintains its original velocity. S_{AB} is constructed in the configuration space, taking into consideration the physical dimensions of each car (represented by the radius of the circumference circles of each car). This is done by reducing car A to a dimensionless point, and enlarging the radius of car B to the sum of the original radii: $A + B$. The gray sector S_A denotes the domain of the absolute velocities of car A that leads to a collision with car B. S_A is a simple linear transformation of S_{AB} along V_B. In Figure 2b, the dashed domain, M_A, denotes the set of available velocities of car A based on its possible maneuvers within the given time horizon Δt. This domain considers the possible accelerating/deceleration, as well as steering maneuvers of car A for the given initial conditions. The dark gray sector in Figure 2b, SM_A, represents all possible avoidance maneuvers of car A that guarantee no collision. This sector is constructed by subtracting the velocity obstacle domain that results in a collision (S_A) from the domain M_A of all possible maneuvers of car A. Point S in Figure 2b denotes a safe avoidance velocity for car A that does not require a change in the car steering, using its maximal speed. If accident avoidance demands maneuvers that are beyond the cars' capabilities, SM_A domain vanishes, and an accident occurs.

The case shown in Figures 2a and 2b involves two cars. However, this algorithm can be easily extended for several cars, by constructing the safe sector as a subtraction of the cars' possible maneuvers domain from all other velocity obstacle domains of all other vehicles (Figure 2c). In their article, Fiorini and Shiller (1998) describe a global search algorithm, as well as heuristics, which constructs a safe trajectory among multiple moving objects using the maximal velocity.

Agents' Vision

SAFEPED agents see the 3D environment within the "view cone" of up to 180° angle, shown as gray sectors in Figure 3a. We interpret the human visual system as a pinhole camera. The 3D shape (currently a minimal 3D box) of each object within the view cone is projected on the retinal plan of the agent's "eye" (Figure 3b).

Each agent detects objects within its view cone and reacts to them according to the behavior model. SAFEPED defines which objects are obscured by others, and to what degree. We use various levels of transparency of agents' bodies to present the obscurity of the agents. Figure 3b presents the scene from the point of view of the driver of the car, marked by the cross in Figure 3a. For entirely obscured agents, only bounding boxes are shown.

Any object that is either fully obscured or beyond an agent's view for a certain time becomes invisible to that agent. From that moment on, these objects do not affect the agent's behavior. This aspect of agents' behavior refers to the organization of the human visual short-term memory (VSTM), which is able to store up to 7 ± 2 items (Miller, 1956). The old items decay over time until they are fully erased. Retention period of the VSTM decreases as the number of items increases, varying from 3 to 20 sec (Peterson & Peterson, 1959). However, some researchers show a reduced retention period of 2 sec (Muter, 1980, 1995). In SAFEPED, we consider the worst case, and assume that the retention period is 2 sec only.

Agents' Behavior Models

Behavior models distinguish between three levels of decision making: strategic, tactical, and operational (Robin et al., 2009). Decisions on motion destinations and motion activities are chosen at the strategic level and are irrelevant for SAFEPED, whose scope is limited to a small area of potential vehicle–pedestrian conflict, such as a crosswalk and its surroundings.

The order of an agent's activities, including route selection, is defined at the tactical level (Robin et al., 2009). SAFEPED generates traffic accident scenarios and the user can draw routes for pedestrians and vehicles, or import route coordinates as an ASCII file. The user assigns agents to the routes and schedules their appearance at the starting point of the route.

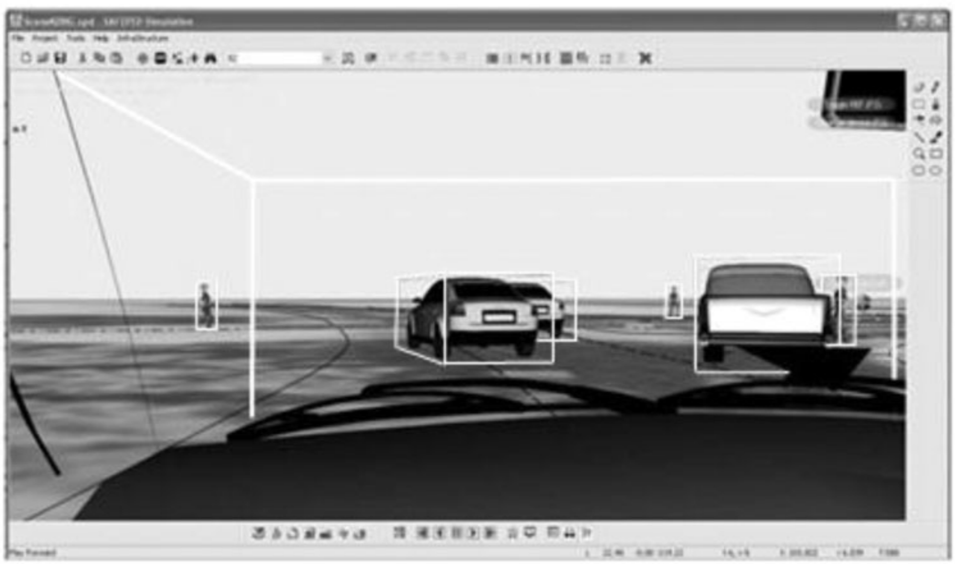
The behavior model of SAFEPED reflects agents' operational decision making during their motion through the area. It consists of two fundamental components:

Steering: A path-following algorithm guides the agents according to Newtonian mechanics, as close as possible to a predefined route. Agents follow collision avoidance algorithms (discussed later) to minimize maneuvers. This is done by minimizing the vector sum of lateral (steering) and longitudinal (braking or acceleration) forces.

Cognition-perception: This component reflects human limitations in sensing the environment, interferes with the steering algorithm, and deteriorates its performance. SAFEPED follows recent research of the drivers' and pedestrians' behavior and allows activating, deactivating, and specifying the following three cognitive-perceptual characteristics:

- *Visibility:* The angle of the agent's view (Atchley & Dressel, 2004; Strayer & Johnston, 2001) is established, as well as the

(a)



(b)

Figure 3 SAFEPED scene of (a) the agents' view cones, where the car marked by a cross is chosen for follow-up, and (b) 3D visibility in the driver's view from the car.

threshold level of an obstacle's visual exposure that is required by the agent in order to identify and classify the object.

- *Short-term visual memory decay*: How long the agent remembers an object that becomes obscured.
- *Time delay in reaction:* The minimal and maximal delay in reaction of drivers and pedestrians to a road situation. The distribution of agents' reaction time within the [min, max] interval is assumed to be uniform.
- *Pedestrian's gap acceptance*: The probability that the pedestrian will cross the road as dependent on the time required for an approaching car to cross the pedestrian's path. This can also depend on the pedestrian's accumulated waiting time (Papadimitriou et al., 2009), age, and the number of pedestrians waiting to cross the road (Wang et al., 2010).

All behavior parameters are defined in SAFEPED for each agent at the individual level.

SAFEPED User Interface and Output

SAFEPED implements Microsoft's Single Document Interface concept, and its scenes serve for establishing the spot's 3D

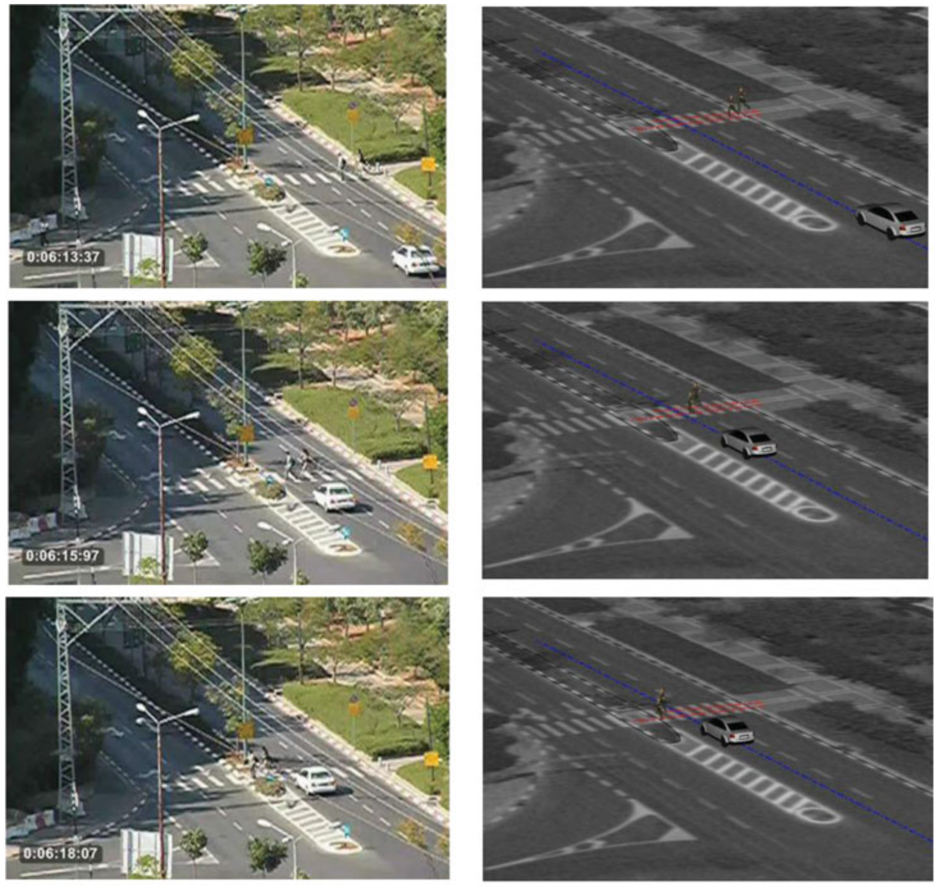


Figure 4 Video snapshot of the road-crossing event (a) versus snapshots of the SAFEPED simulation (b) taken at the same moments.

surface model and the agents' trajectories, which can be edited using the SAFEPED graphical user interface (GUI):

- *Surface* can be imported into SAFEPED in the Microsoft Direct X (*.x) format. Usually, a surface is presented by the Digital Terrain Model (DTM) and overlapping orthophoto.
- *Drivers' and pedestrians' trajectories* can be drawn within the SAFEPAD or imported in an ASCII format. Usually, these are planned and drawn for creating potential accident situations.
- *Parked vehicles* are placed along the agents' trajectories with the help of the SAFEPED tool. The user can select the type of parked vehicle from a list of available models: buses, trucks, minivans, minibuses, private cars, and motorcycles.

Agents' actions are recorded at every time step, and can be analyzed and replayed. The model keeps track of an agent's location, set of available velocities, eye-sight behavior, decisions on velocity, distance to other agents, and acceleration. All types of accidents between the agents (i.e., head-on collision, one-sided collision, car-pedestrian collision, etc.) are also registered. The performance of SAFEPED remains very high in all experiments, in which up to a hundred of simultaneously moving agents were simulated. The movies that present the general view of the pilot version of SAFEPED, formalization of visibility, and traffic accidents are presented on YouTube.[2]

SAFEPED VALIDATION AND VERIFICATION

To validate SAFEPED, we compare the performance of the real and simulated agents in a road-crossing event in which two pedestrians cross the road in front of an approaching car. The comparison is based on analysis of a video record of the event. For SAFEPED verification, we qualitatively compare the performance of agents in two simulated scenarios in which the driver agents, subject to road constraints, must react to crossing pedestrians in order to prevent a collision.

[2]See http://www.youtube.com/watch?v=ia3W8oiTVYw&feature=related, http://www.youtube.com/watch?v=6KFcfFRElt8&feature=related, and http://www.youtube.com/watch?v=axWEGNetpM0

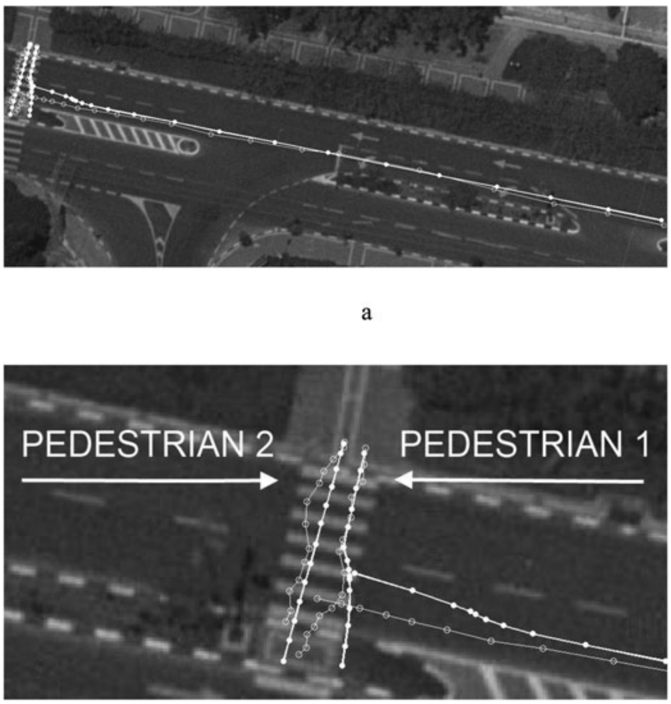


Figure 5 Distant (a) and close (b) view of the trajectories of pedestrians and the car extracted from the video clip (gray) and simulated by the SAFEPED (white).

SAFEPED Validation Using Real-World Video Records

Validation of SAFEPED is based on a comparison between real and simulated movement of drivers and pedestrians. The real movement is represented by a series of video clips recorded by two synchronized cameras—close-range for pedestrians and far-range for vehicles. The data are recorded at a frequency of 2 Hz (every 0.5 sec). A network of control points acquired with the Leica GPS9000 RTK Survey System (0.02 m accuracy) is used for transformation of object coordinates in the video frame to a world coordinate system. The road-crossing event (Figure 4) is modeled in SAFEPED using high-resolution (0.1 m) orthophoto. The data extracted from the video include time stamp and world coordinates of the pedestrians and the car that are further used for calculating agents' speeds.

In the simulation, the agents start moving at the same moment and at the same speeds as in reality. From then on, the simulated agents react according to SAFEPED's models, and the car reduces its speed before the crosswalk while pedestrians complete crossing. The real and simulated trajectories of the car and two pedestrians are presented in Figure 5.

Locations and speeds of the car and pedestrian as calculated based on the video clips were then compared to data generated by SAFEPED. Table 1 summarizes the results of this comparison.

Let us analyze the differences between the trajectories in more detail. In the following, the trajectories are compared

Table 1 Statistics of the difference between the location and speed of the agents at the same time as recorded in the video and simulated by the SAFEPAD.

	Difference in location (m), average and STD	Difference in speed (m/sec), average and STD
Car	1.94 (1.44)	0.51 (0.66)
Pedestrian 1	0.79 (0.60)	0.75 (0.67)
Pedestrian 2	0.73 (0.34)	0.56 (0.27)

starting from the moment an agent commences moving and ending at the moment the pedestrians and the car do not react to each other anymore: seconds 9.0–17.5 for the car, and seconds 11.5–17.5 and 12.0–17.5 for Pedestrian 1 and Pedestrian 2, respectively.

The Car

According to Figure 6, the real driver reacts to the pedestrians about 1 sec before the simulated driver. The SAFEPED driver agent behaves more smoothly than the real one and does not accelerate in the beginning of the period (second 10.0), before both real and simulated driver slow down. Different from reality, a SAFEPED driver drives essentially more slowly during the slow phase of driving, seconds 14.5–16.5.

The average difference in locations, at the same moment of time, between the real and simulated cars is 1.94 m (STD = 1.44 m), and varies between 0.5 and 3.3 m. The time that is necessary to cover this distance is between 1 and 2 sec for the slowest part of the trajectory (seconds 15.5–17.0), while for the higher speeds this time is below 0.5 sec, lower than the full driver's reaction time of 0.7 sec (Green, 2000).

Pedestrians

The real and simulated trajectories of Pedestrian 1 are very close during the first half of the event and diverge, having similar bending, during the second part (Figure 5b). The first pedestrian is closer to the approaching car than the second one and is forced to maneuver urgently, while the second pedestrian goes behind the first one and reacts less impulsively. As shown in Figure 7, both in the video and in simulation, Pedestrian 1 accelerates to a speed of 1.8 m/sec at the beginning of the trajectory, and then slows down to 0.5–1 m/sec. Model Pedestrian 1 slows down at second 13.5, 1.5 sec before the real pedestrian (second 14.0). The average difference in locations is about 0.8 m (STD = 0.6 m).

According to the video record, Pedestrian 2 follows Pedestrian 1 (Figure 8). This line of action does not require excessive maneuvering and results in high similarity of captured and simulated trajectories (Figure 5b). The average difference in locations is 0.73 m (STD = 0.34 m).

To summarize, the agents' movements in the SAFEPED simulation fit well to the reality, while presenting some differences that can be attributed to the lack of "human" properties, such

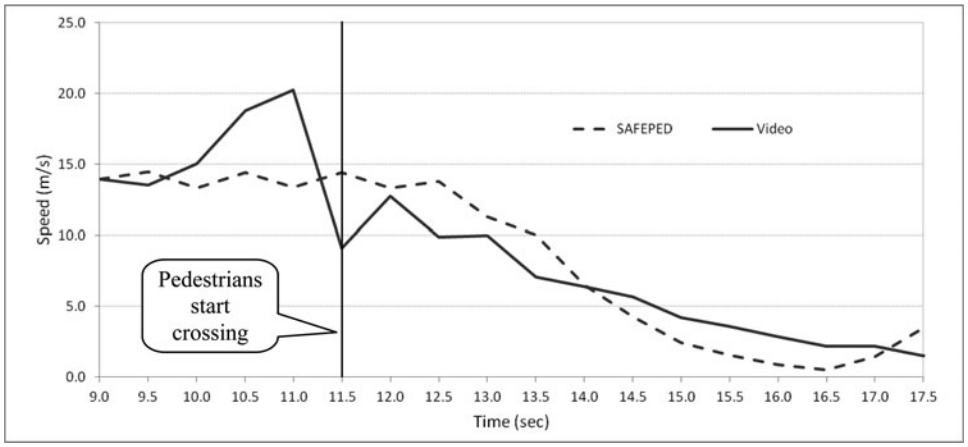


Figure 6 The speed of the car in reality and in the SAFEPED simulation.

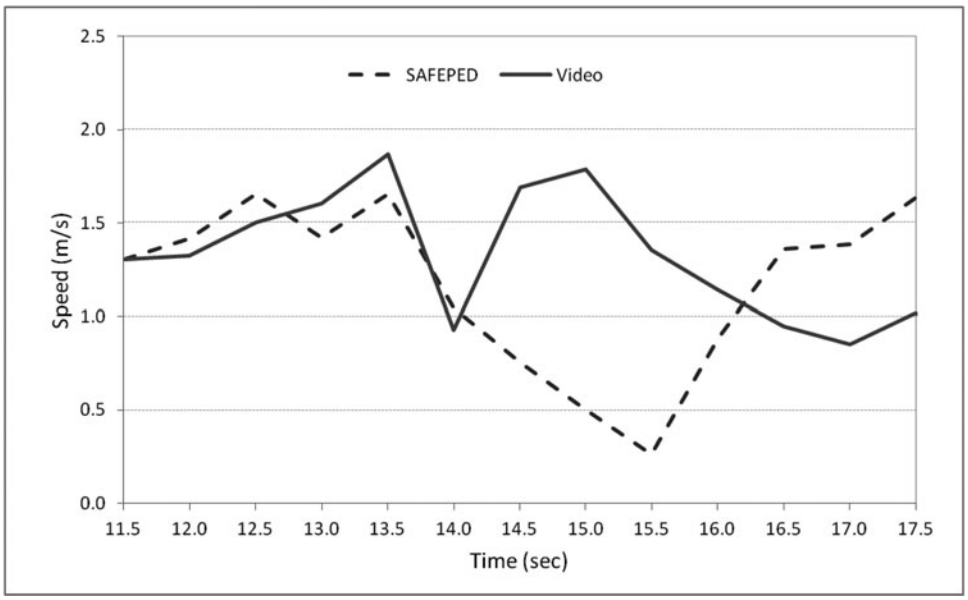


Figure 7 The speed of Pedestrian 1 in reality and in the SAFEPED simulation.

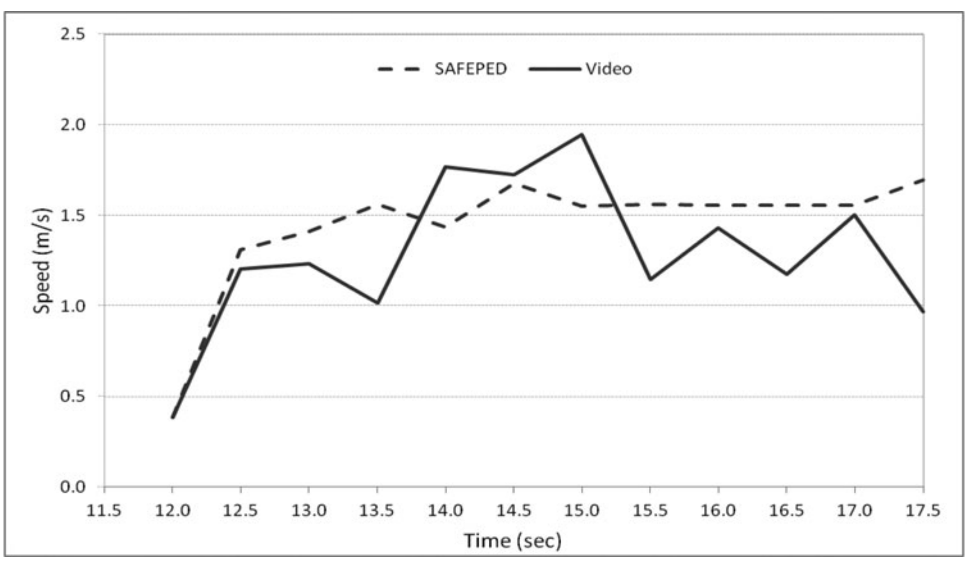


Figure 8 The speed of Pedestrian 2 in reality and in the SAFEPED simulation.

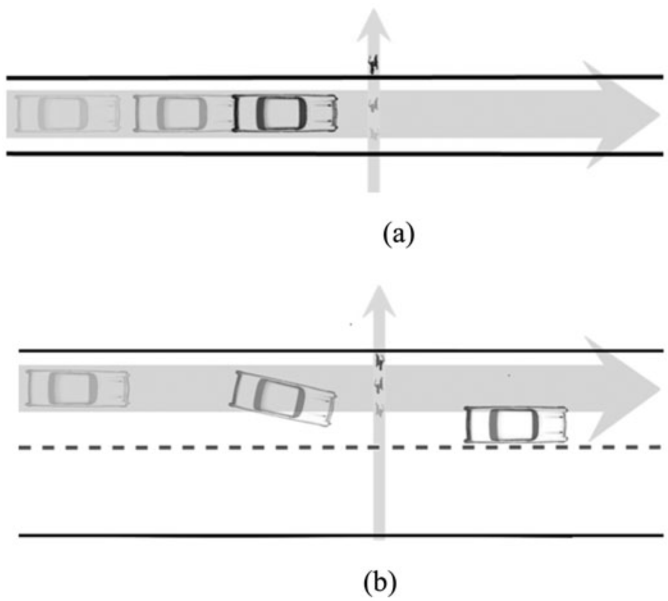


Figure 9 One-lane (a) and two-lane (b) scenarios.

as driver's caution or pedestrian unwillingness to walk quickly. These properties can be estimated based on the analysis of the video clips, and this will be the next step of the SAFEPED validation.

Qualitative Verification

We qualitatively verified SAFEPED by simulating potential vehicle–pedestrian accident scenarios in which the driver noticed the pedestrian crossing the road shortly before the potential collision. In our experiments, if the driver does not react, the car will inevitably hit the pedestrian.

In the first series of simulations, we consider a one-lane narrow road where the driver is not able to steer and the accident can be avoided only by braking (Figure 9a). In the second scenario, the car drives along a two-lane road and can brake and steer (Figure 9b). The pedestrian crosses perpendicularly to the road at a speed of 3.5 km/h and does not react to the approaching vehicle. The driver detects the crossing pedestrian 1.6 sec or less before the potential crash (alert time).

In our scenarios, the car's initial speed varies from 50 to 20 km/h and alert time between 1.6 and 0 sec. According to the 100-Car Naturalistic Driving Study (Klauer et al., 2006a) we limit the car's acceleration/deceleration in our scenarios to 6.0 m/sec^2.

Tables 2a and 2b (one-lane road) and 3a and 3b (two-lane road) show the minimal distance between the car and pedestrian (zero value represents a collision) and the minimal speed of the car observed during the simulation in the two scenarios.

According to the results of the first experiment (Tables 2a and 2b), there is a strong correlation between the car's speed and the minimal time alert time for the driver to avoid the accident. As in reality, the driver stops or slows down on identifying the pedestrian, depending on the distance to the pedestrian and the minimal distance between the car and pedestrian is about 1.5–2 m (Table 2a), regardless of the initial car speed. Note that according to the conditions of the experiment, cars moving more slowly identify the pedestrian closer to the point of potential collision in comparison to cars moving more quickly. Consequently, cars moving more slowly (20–25 km/h) always stop in order to avoid a collision, while the drivers of the cars moving more quickly have sufficient time to decelerate and let the pedestrian cross the road, without stopping the car (Table 2b).

The second experiment demonstrates that on a wider road, where the driver can steer, the minimal alert time necessary to avoid the accident is 0.4–0.5 sec (Table 3a), shorter than in the case of the single-lane street (Table 2a). Moreover, we can see that steering is the driver's last-choice solution in the case of a short alert time only. Indeed, no matter what the initial speed of the car is, for the longer alert times, say up to 1.3 sec in case of an initial speed of 50 km/h, the driver brakes and stops (Table 3b). The driver only steers for the shorter alert times. Note that steering helps to avoid the accident for the alert times that would inevitably result in a collision on the narrow road. In case of an initial speed of 50 km/h, this additional interval is 1.1–0.8 sec. Just as in the scenario of narrow road, the drivers of cars moving more slowly identify pedestrians crossing the road closer to the point of possible collision, and their ability to avoid a collision by steering is limited in comparison to the drivers of cars that initially move more quickly.

Table 2a Simulated scenario 1, minimal distance (m) between the car and the pedestrian.

		Alert time (sec)																
		1.6	1.5	1.4	1.3	1.2	1.1	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
Car speed (km/h)	50	1.63	1.61	1.86	1.62	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	45	1.66	1.60	1.64	1.67	1.55	1.35	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	40	1.64	1.65	1.68	1.64	1.77	1.63	1.18	0.28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	35	1.67	1.68	1.70	1.64	1.69	1.56	1.73	1.21	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	30	1.46	1.68	1.99	1.82	1.64	1.65	1.56	1.64	1.25	0.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	25	1.49	1.66	1.67	1.58	1.70	1.66	1.66	1.57	1.51	1.07	0.24	0.00	0.00	0.00	0.00	0.00	0.00
	20	1.45	1.66	1.77	1.65	1.65	1.56	1.33	1.26	1.54	1.38	0.80	0.36	0.00	0.00	0.00	0.00	0.00

Table 2b Simulated scenario1, minimal speed (km/h) of the car.

		Alert time (sec)																
		1.6	1.5	1.4	1.3	1.2	1.1	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
Car	50	2.5	2.4	0.0	0.0	0.0	0.2	15.9	21.4	26.9	29.7	34.3	37.0	39.8	42.6	45.3	47.2	50.0
speed	45	2.7	2.7	2.8	0.0	0.0	0.0	0.0	13.6	19.1	24.7	27.4	32.0	33.9	37.6	39.4	43.1	45.0
(km/h)	40	2.3	1.8	2	1.9	0.0	0.0	0.0	0.0	13.2	17.8	23.4	26.1	29.8	32.6	35.3	37.2	39.9
	35	1.7	0.0	1.3	2.1	0.0	0.0	0.0	0.0	0.0	11.0	15.6	21.1	23.9	27.6	29.4	32.2	34.9
	30	3.0	0.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.8	15.2	18.0	22.6	24.4	28.1	29.9
	25	1.7	0.0	0.9	1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.4	13.9	16.7	20.3	22.2	24.9
	20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.0	10.7	14.4	17.2	20.0

CASE STUDY OF A DANGEROUS ROAD SITUATION

To test SAFEPED we investigate one of the most hazardous scenarios at a crossroad: A high vehicle obscures part of the road from the pedestrian who has started crossing. An essential part of the road is hidden from the pedestrian's view, while an essential part of the sidewalk, including the starting point of the crosswalk, is hidden from the driver's view. A U.S. Transportation Agency publication (Hunter et al., 1997) describes this situation as follows. "The pedestrian entered the traffic lane at midblock in front of standing or stopped traffic and was struck by another vehicle moving in the same direction as the stopped traffic." According to Zegeer et al. (2005), multiple-threat crashes comprise 17.6% of pedestrian collisions on marked crosswalks.

Experimental Setup

The experimental setup is presented in Figure 10: High truck A stops in the right lane of a two-lane road, and the pedestrian's view of the road is obscured. No traffic lights are installed. The pedestrian decides to cross the street, based on her own estimate of the traffic conditions. The pedestrian is obscured from the view of the driver of Vehicle B approaching the crosswalk along the left lane. If the pedestrian and the car continue with their current velocities and trajectories, an accident will occur at a point marked by the star (point C).

A real road crossing was chosen for the 3D representation of junction infrastructure, and we investigate the accident situations for three different locations of the obscuring truck: at a distance of 0.75, 2.25, and 3.75 m from the crosswalk (Figure 11).

We investigate the risk of contact between a car in the lane adjacent to the truck and the pedestrian (the pedestrian being hit by the car's right fender) (Figure 12), as dependent on the velocities and the reaction/attention times of the driver and the pedestrian. According to Hoogendoorn et al. (2005), we set the pedestrian reaction time as 0.28 ± 0.07 sec and the driver reaction time as 0.70–0.75 sec (Green, 2000). The reaction times include all possible components that affect the reaction, among them 0.2 sec required to lift the foot from the accelerator and touch the brake pedal. We use $0.28 - 0.07 = 0.21$ sec as the reaction time of a slowly reacting pedestrian and $0.28 + 0.07 = 0.35$ sec as the reaction time of a quickly reacting pedestrian.

Individual Accident Avoidance

Let us investigate the conditions in which the pedestrian or the driver is capable of avoiding a collision, even if the other participant chooses the worst line of action.

We start with an inattentive pedestrian who does not look around and start crossing the street at a high speed of 6 km/h (Table 4). For the situation where the truck is stopped 0.75 m from the crosswalk, the attentive car driver identifies the pedestrian and stops safely when the car's speed is lower than 12 km/h for the slowly reacting pedestrian, and 13 km/h for the quickly reacting pedestrian. When the truck is stopped 2.25 and 3.75 m

Table 3a Simulated scenario 2, minimal distance (m) between the car and the pedestrian.

		Alert time (sec)																
		1.6	1.5	1.4	1.3	1.2	1.1	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
Car	50	1.5	1.5	1.5	1.6	0.8	0.8	0.8	0.8	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
speed	45	1.6	1.6	1.4	1.5	1.6	1.0	0.9	0.9	0.8	0.8	0.7	0.0	0.0	0.0	0.0	0.0	0.0
(km/h)	40	1.6	1.6	1.5	1.5	1.5	1.5	1.0	0.9	0.9	0.8	0.8	0.0	0.0	0.0	0.0	0.0	0.0
	35	1.6	1.7	1.7	1.6	1.5	1.6	1.4	1.2	1.2	0.8	0.8	0.8	0.8	0.0	0.0	0.0	0.0
	30	1.6	1.5	1.6	1.5	1.8	1.8	1.6	1.4	1.9	1.7	1.7	1.5	0.7	0.0	0.0	0.0	0.0
	25	1.5	1.6	1.6	1.6	1.5	1.6	1.5	1.6	2.0	1.7	1.4	1.3	0.0	0.0	0.0	0.0	0.0
	20	1.5	1.5	1.8	1.6	1.4	1.4	1.3	1.3	1.4	1.1	1.0	0.4	0.0	0.0	0.0	0.0	0.0

Table 3b Simulated scenario 2, minimal speed (km/h) of the car.

		Alert time (sec)																
		1.6	1.5	1.4	1.3	1.2	1.1	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
Car speed (km/h)	50	4.4	3.6	5.0	0.0	31.6	32.5	32.5	33.4	34.4	29.7	34.3	37.1	39.9	42.6	45.4	47.2	50.0
	45	4.0	3.8	6.6	3.7	0.0	28.4	28.4	29.4	30.3	30.4	33.0	32.1	33.9	37.6	39.5	43.2	45.0
	40	2.8	2.2	4.0	2.9	0.9	0.0	24.3	25.8	26.2	27.1	28.7	26.2	29.9	32.6	35.4	37.2	40.0
	35	2.0	1.2	1.0	1.5	0.9	0.0	0.0	22.1	22.1	23.0	24.0	25.8	24.5	27.6	29.5	33.2	35.0
	30	2.3	2.1	1.2	0.9	0.0	0.0	0.0	3.2	20.8	19.9	21.4	20.0	21.7	22.6	24.5	27.2	30.0
	25	2.0	1.5	1.1	1.1	1.0	0.0	0.0	0.0	18.6	18.6	19.5	18.6	13.0	16.7	20.4	22.2	25.0
	20	2.1	0.9	0.0	0.0	1.9	0.8	0.0	0.0	0.0	9.6	12.9	0.0	7.1	10.8	15.4	17.2	20.0

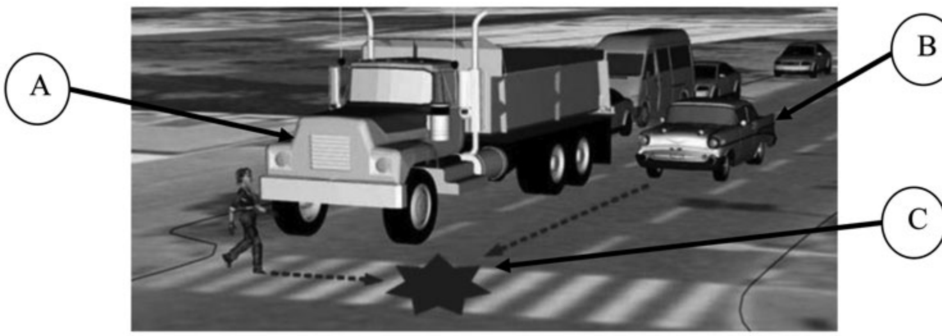
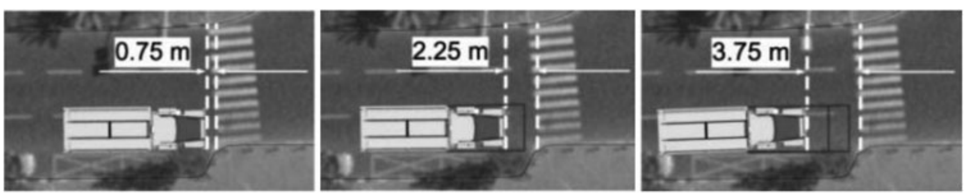


Figure 10 Experimental setup: High truck A stops at the side of a crosswalk and obscures the view of both the approaching driver (B) and the walking pedestrian (C).



(a) (b) (c)

Figure 11 Three experimental situations: The truck stops at a distance of 0.75 m (a), 2.25 m (b), and 3.75 m (c) from the crosswalk border.

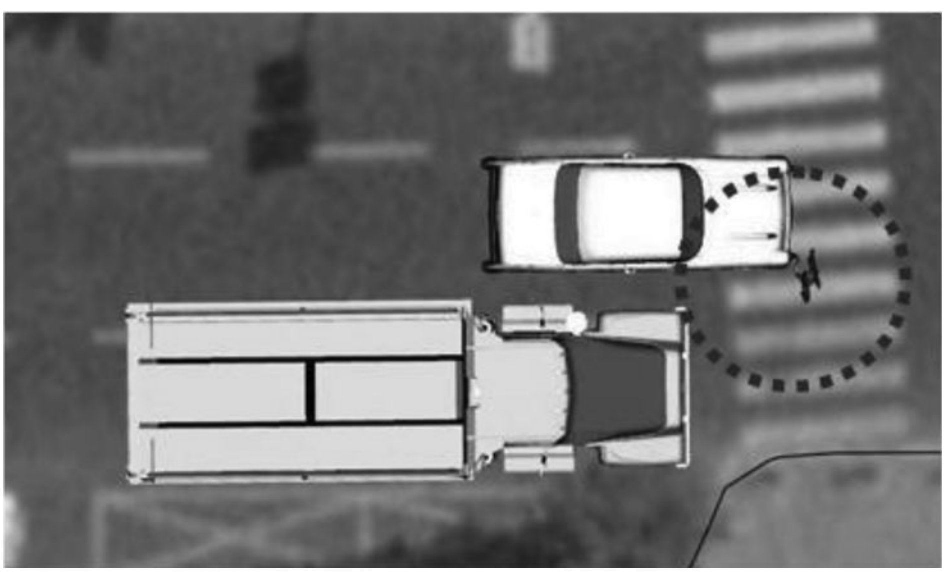


Figure 12 Car's right fender hits pedestrian when truck is parked 0.75 m from the crosswalk.

Table 4 Maximal safe vehicle speed for attentive driver and inattentive pedestrian crossing the road at a speed of 6 km/h as dependent on the distance between the obscuring truck and the crosswalk.

	Distance between the obscuring truck and the crosswalk		
Pedestrian's reaction	0.75 m	2.25 m	3.75 m
Slow	12 km/h	24 km/h	26 km/h
Fast	13 km/h	25 km/h	28 km/h

from the crosswalk, the car driver succeeds in preventing the accident if the car's speed is below 24–28 km/h.

The results for the case of an inattentive driver who drives at a speed of 50 km/h and an attentive pedestrian who slows down when crossing a road are presented in Table 5. To avoid an accident, in case of a truck stopped at 0.75 m, the velocity of the slowly reacting pedestrian should be below 3.7 km/h, while the velocity of the quickly reacting pedestrian should be below 4.6 km/h (Table 5). When the truck is stopped further from the crosswalk, the pedestrian is capable of detecting the car at any reasonable walking speed.

The Situation in Which Both Participants Have to React

Figure 13 presents the relation between maximal safe speeds of the car and the pedestrian for inattentive and attentive agents when the obscuring truck is stopped 0.75 m from the crosswalk.

Table 5 The maximal safe speed of attentive pedestrian in the case of an inattentive driver moving at a speed of 50 km/h.

	Distance between truck and crosswalk		
Pedestrian's reaction	0.75 m	2.25 m	3.75 *m*
Slow	3.7 km/h	5.1 km/h	Above 6.0 km/h
Fast	4.6 km/h	6.0 km/h	Above 6.0 km/h

As can be seen from the chart, the pedestrian's reaction is critically important for avoiding the accident. The slowly reacting attentive pedestrian will be in danger if the car's speed is above 20 km/h, while for the quickly reacting pedestrian the dangerous speed is above 35 km/h. Note that in order to avoid a crash regardless of the car's speed, the pedestrian should not walk faster than 2 km/h.

Tables 4 and 5 and Figure 13 are based on estimating the moment of physical contact between the pedestrian and the car. Safety recommendations require significant nonzero distance between the car and pedestrian during the entire period of their interaction. In order to meet these requirements, we must include essential margins to the estimates presented in Tables 4 and 5 and in Figure 13. Let us estimate these margins.

Margins for Safe Avoidance of Crash

We define a road situation as "safe" if the driver and the pedestrian avoid an accident by passing next to each other at a distance of 0.5 m, and present the case of the truck stopped at 0.75 m only, and the worst case for the slowly reacting pedestrian. As can be seen from Table 6, the safe speeds are essentially lower than obtained in the previous section.

Our results can be compared to the results of the Pedestrian Safety Countermeasure Deployment Project—a 6-year field study of the effectiveness of different safety installations at pedestrian crossroads, depending on the crosswalk's potential for vehicular–pedestrian collision (Redmon, 2011). Among other modifications, the advanced stop line, marked 4–10 feet (1.2–3.0 m) before the crosswalk and aimed at discouraging motorists from entering the crosswalk, was tested at 14 San Francisco intersections. The effectiveness of this line was estimated based on video records and interviews with pedestrians, and the authors conclude that wider study is necessary concerning the stop-line effectiveness. Our simulations add to this study and are not affected by the interviewees' subjective experience: The advanced stop line is more effective if marked even further away—at 3.5 m before the crosswalk, which

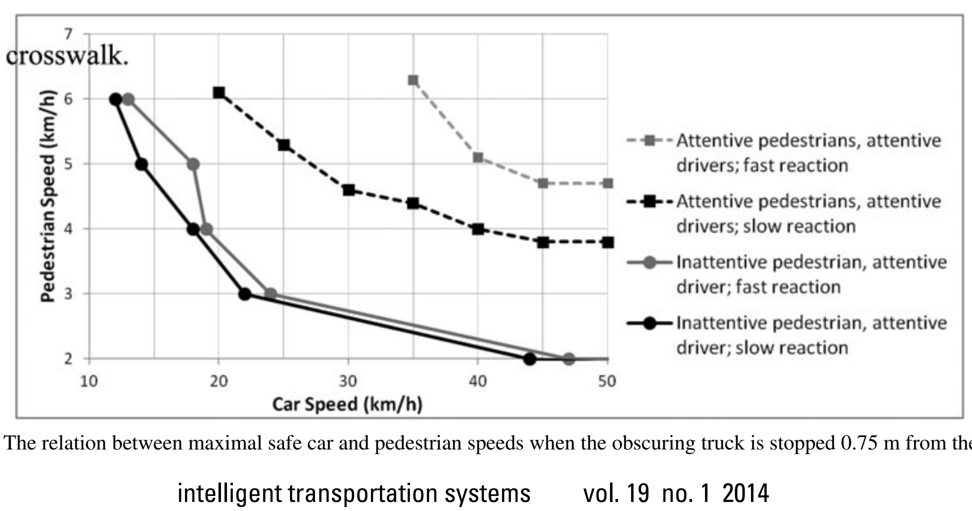


Figure 13 The relation between maximal safe car and pedestrian speeds when the obscuring truck is stopped 0.75 m from the crosswalk.

Table 6 Minimal distance between car and slowly reacting pedestrian for the truck stopped 0.75 m from the crosswalk.

	Car's speed, km/h						
Pedestrian's speed, km/h	50	45	40	35	30	25	20
5.5	crash	crash	crash	crash	crash	crash	0.17
5.0	crash	crash	crash	crash	crash	0.06	0.16
4.5	crash	crash	crash	crash	0.10	0.19	0.29
4.0	crash	crash	crash	0.08	0.08	0.26	**0.36**
3.5	0.04	0.04	0.06	0.10	0.18	0.28	**0.47**
3.0	0.10	0.09	0.11	0.20	0.26	0.36	**0.45**
2.5	0.22	0.20	0.47	0.71	1.05	1.08	**0.47**
2.0	0.94	1.14	1.00	0.99	1.01	1.02	**0.42**

Note. "Crash"—zero distance, shaded cells—unsafe speeds (distance between the car and pedestrian below 30 cm—half the width of the human body); boldface—pedestrian can avoid accident on his or her own.

is greater than the maximal distance investigated in Redmon (2011).

SUMMARY

We presented SAFEPED—a multi-agent, microscopic 3D simulation of car and pedestrian dynamics at black spots. By direct assignment of human-based behavioral rules to the model agents, SAFEPED is capable of arbitrarily implementing cognitive-perceptual parameters of driver and pedestrian behavior, including strategic and tactical behavioral components. The high temporal and spatial resolution of SAFEPED, similar to those of driver simulators and real-time in-car equipment, provides high potential for combining it with data from field studies (Kiefer et al., 2003; Klauer et al., 2006a, 2006b; Najm & Smith, 2004; Smith et al., 2003). SAFEPED can serve as a tool for assessing accident risks at specific spots, and can identify safety measures to minimize these risks.

SAFEPED can serve as a sophisticated tool for assessing modifications to existing and hypothetical black spots, identifying those safety countermeasures that will provide maximum road safety benefits. The model does not propose safety measures, but estimates the results of their implementation. For example, our model can help in testing the usefulness of the safety measures and in establishing priorities in their implementation.

Future developments of SAFEPED include adding more complex behavior models for agents, allowing for factors such as age, alcohol consumption, physical disabilities, and so on, and examining their effect on agents in black spots. SAFEPED can also be applied to road safety research on specific hazardous spots, such as crosswalks near schools, allowing researchers to see the scene from the viewpoint of a child and to examine the effect of various parameters on children's behavior in crosswalks.

ACKNOWLEDGMENT

The authors are grateful to Eilon Blank for taking and analyzing video clips that are used in this article.

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